# Exam 3

Your Name:

## Instructions

Solve each of the following problems to the best of your abilities. The exam is worth 100 points total and is calibrated for 90 minutes. You must show all your work to receive full credit.

Once you have completed the exam, hand it to me, and you can take a break before the second part of class. Class resumes at 2:00PM.

Good luck!

# Problem 1

(15 points) In class, we went over many examples where we calculated the magnetic field in and around a cylindrical wire with some radius R. Modeling a wire with current uniformly distributed over its circular face works well for DC circuits, but it does not accurately represent AC current moving through a wire. In the case of AC circuits, the current is concentrated at the surface of the wire with no current in the body of the wire itself. For example, in the diagram below, the current density is located along the outer surface of the wire in the dark orange section, but not in the interior of the wire in the pale orange section.

Consider a cylindrical wire with a radius R. This wire carries a very thin layer of current density at the surface of the wire given by $J = \frac{I}{2\pi R}$. Use Ampere's law to solve the following problems. Show all of your work!

a) (5 points) What is the magnitude of the magnetic field at the center of the wire?
b) (5 points) What is the magnitude of the magnetic field when you are halfway between the center of the wire and its radius R?
c) (5 points) What is the magnitude of the magnetic field when you are a distance r = 2R from the center of the wire?

# Problem 2

(25 points) A uniform magnetic field B = 0.5 T is oriented in the +y direction. A positively charged particle with charge q = 0.6 mC and mass m = 0.02 kg is launched into the magnetic field with an initial velocity of v = 150 m/s in the +z direction. The charged particle undergoes uniform circular motion due to the magnetic field.

a) (5 points) Draw a picture of the setup. Be sure to clearly label the magnetic field, the particle, the initial velocity of the particle, the circular path of the particle, and the direction that the particle takes along the path as it moves through space.
b) (5 points) What is the magnitude of the centripetal force acting on the particle?
c) (5 points) What is the frequency of the oscillations of the particle?
d) (5 points) What is the net work done on the particle by the magnetic field as it makes one complete revolution in the field?
e) (5 points) Suppose I were to reverse the direction of the magnetic field. Would the radius of the circular path increase, decrease, or stay the same? Why?

## Problem 3

(25 points) The magnetic field in some region of space is given by the following expression (with units of Teslas):

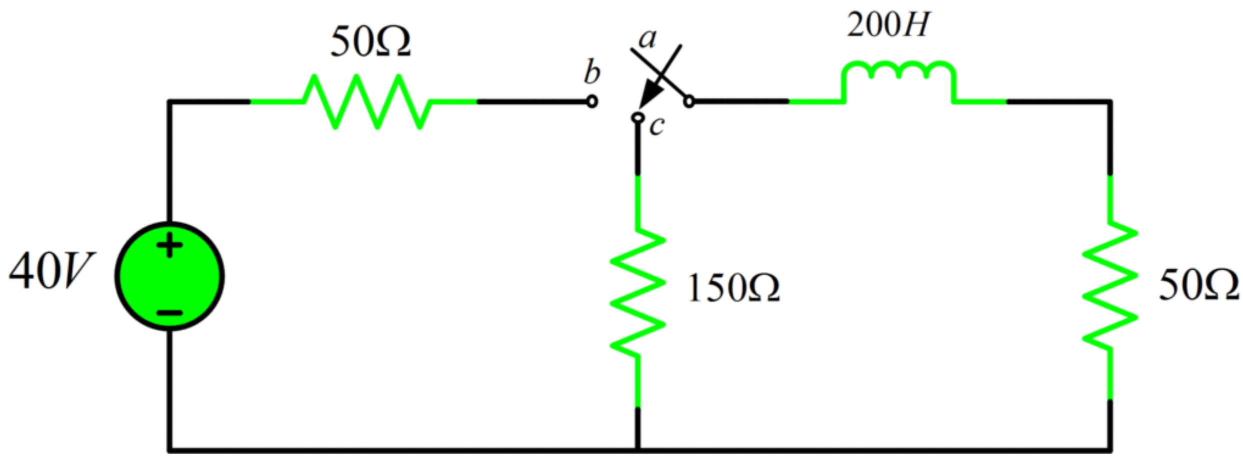
$$\vec{B}(t) = 10.7 \cos\left(\pi t + \frac{\pi}{4}\right) \hat{z}$$

A square loop of wire with an area of 4.9 square meters and a total resistance of 100 ohms is positioned in the xy-plane and centered at the origin. Assume that the area vector of the loop points in the **-z direction**.

a) (5 points) Draw a graph of the magnetic flux through the loop as a function of time. Calculate and label (1) the period of the graph and (2) the amplitude of the graph.
b) (5 points) Draw a graph of the induced EMF in the loop as a function of time. Calculate and label (1) the period of the graph and (2) the amplitude of the graph.
c) (5 points) What is the direction of the current in the loop at time t = 0 seconds? Assume that you are on the positive z-axis looking down at the loop.
d) (5 points) Solve for the times when the current in the loop is equal to zero.
e) (5 points) Suppose I double the amplitude of the oscillating magnetic field given above. Would the amplitude of the induced EMF in the loop double as well? Why or why not?

## Problem 4

(20 points) A charging / discharging RL circuit is constructed as shown below. When the switch is connected between a and b, the battery charges the inductor through two of the three resistors. When the switch is connected between a and c, the inductor discharges through a different pair of resistors.




a) (5 points) Suppose that the switch is connected between points a and b. What is the the current flowing in the inductor after a long time (magnitude and direction)?
b) (5 points) Suppose that the switch is connected between points a and c after the inductor has had a chance to become fully charged by the 40 V battery. What is the current flowing in the inductor immediately after the connection between a and c is made (magnitude and direction)?
c) (10 points) Derive an expression for the current in the circuit as a function of time when the switch is connected between points a and c. You can assume that the inductor was fully charged by the battery before the switch was connected between points a and c. Show all of the work of your derivation, starting from Kirchhoff's laws.

# Problem 5

(15 points) In class, we went over an example of how the Hall Effect was used to determine the sign of charge carriers in a material. In our example, it was used to determine that negatively charged electrons are the charge carriers in a metal wire – not positive charges.

However, in some materials, both positive and negative charges carry currents simultaneously. For example, if you prepare a saltwater solution and pass a current through it, the positive sodium and negative chloride ions in the solution will move in opposite directions. Both of those effects contribute to the final current.

a) (5 points) Explain how the Hall Effect experiment might change if positive and negative charges were flowing simultaneously in opposite directions. Use the concepts that we discussed in the magnetism unit and the right-hand rules to defend your explanation.
b) (5 points) Draw a diagram to support your explanation.
c) (5 points) Do you think the relative speeds of the charges make a difference? For example, would it matter if sodium ions move faster than chloride ions in the saltwater example above? Why or why not?

## ✸ Extra Credit ✸

(10 points) Think about the setup of Problem 1. Why do you think that the model for DC circuits is different than that for AC circuits? Be specific in your answer, citing any relevant laws of nature.