Exam 2

Your Name:

Instructions

Solve each of the following problems to the best of your abilities. The exam is worth 100 points total and is calibrated for 90 minutes. Be sure to show all of your work for full credit.

Once you have completed the exam, hand it to me, and you can take a break before the second part of class. Class resumes at noon.

Good luck!

Problem 1

(25 points) The electric potential in a region of space is given by the equation:

$$V(x, y) = 5(x^2 - y^3) - 10$$

a) (5 points) What is the magnitude of the electric potential difference between (2.0 m, 1.0 m) and (3.0 m, 4.0 m)?
b) (5 points) What is the electric field at the point (3.0 m, 4.0 m)? Remember that electric field is a vector!
c) (5 points) Suppose I place a -100 mC charge at the position (2.0 m, 1.0 m). What is the electric force on the particle? Remember that electric force is a vector!
d) (5 points) Suppose a +50 mC charge starts at position (2.0 m, 1.0 m) and moves to the origin. What is the change in electric potential energy of this particle?
e) (5 points) Suppose I were to introduce a DC offset of +10 V to the graph. In other words, I shifted the entire graph up by 10 V. How would that affect the change in potential energy on the particle that you calculated in part (d)? Explain your reasoning.

Problem 2

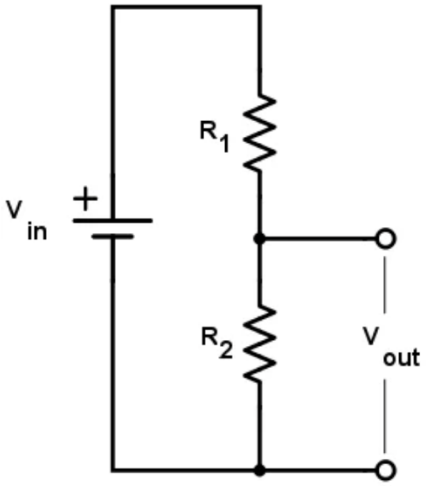
(15 points) Two protons start very far apart and are launched towards each other, each with a velocity of 10,000 m/s.

- The charge on a proton is $+1.6 \times 10^{-19}$ C
- The charge on an electron is -1.6×10^{-19} C
- The mass of a proton is 1.67×10^{-27} kg
- The mass of an electron is 9.11×10^{-31} kg

a) (5 points) What is the closest distance that the protons will get in their approach?
b) (5 points) What is the total electric potential energy of the system when the protons are at their closest distance?
c) (5 points) An electron volt (eV) is a unit of energy defined as the amount of kinetic energy gained or lost by a single electron when it accelerates through an electric potential difference of one volt. How many electron volts is your answer in part (b)?

Problem 3

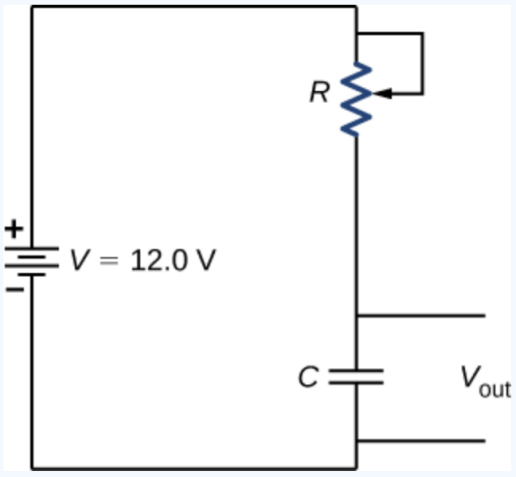
(20 points) A two resistor voltage divider is a circuit used to create a voltage less than or equal to some input voltage. In the diagram below, V_{in} is the input voltage, R_1 and R_2 are resistors that you use to tune the output voltage, and V_{out} is the output voltage. In other words, the output voltage is just the voltage across R_2.



a) (5 points) Derive an expression for V_{out} in terms of R_1, R_2, and V_{in}.
b) (5 points) Derive an expression for the power dissipated in resistor R_2 in terms of R_1, R_2, and V_{in}.
c) (5 points) What value of R_1 should I choose if I want V_{out} to equal V_{in}? Explain your reasoning.
d) (5 points) Suppose I were to increase the value of R_1 while keeping V_{in} and R_2 the same. Would you expect V_{out} to increase, decrease, or stay the same? Why?

Problem 4

(15 points) A relaxation oscillator is used to control a pair of windshield wipers. The relaxation oscillator consists of a 9.5 mF capacitor and a variable resistor known as a rheostat. A knob connected to the variable resistor allows the resistance to be adjusted from zero to 10,000 ohms. The output of the capacitor is used to control a voltage-controlled switch. The switch is normally open, but when the output voltage reaches 7.0 V, the switch closes, energizing an electric motor and discharging the capacitor. The motor causes the windshield wipers to sweep once across the windshield and the capacitor begins to charge again from empty. To what resistance should the rheostat be adjusted for the period of the wiper blades to be 8.0 seconds?



Problem 5

(25 points) An insulating cylindrical shell with an inner radius of A and an outer radius of B carries a uniform change density of ρ (where ρ has units of charge per unit volume). We are going to calculate the electric field inside of the shell using Gauss' law.

$$\Phi_{Closed} = \frac{q_{enc}}{\epsilon_o}$$

a) (5 points) Sketch a diagram showing the cylindrical shell and the Gaussian surface that you will use to calculate the electric field. Remember that we want to find the electric field at a point inside of the shell. Be sure to label any relevant dimensions of the Gaussian surface.
b) (5 points) The left side of Gauss' law (shown above) describes the net electric flux through the closed surface. Set up an expression for the net flux and simplify it using the diagram that you drew as a reference. Remember to think about symmetry to help you simplify the expression!
c) (5 points) The right side of Gauss' law (shown above) describes the net charge enclosed by the closed surface. Set up an expression for the net charge and simplify it using the diagram that you drew as a reference.
d) (5 points) Combine your answers for parts (b) and (c) to get an expression for the electric field inside of the shell.
e) (5 points) Suppose the charge density in the shell was not uniform. Instead, it had a dependence on the distance from center of the wire: $\rho(r) = Ae^{-kr}$ where A and k are positive constants and r is the distance from the center of the wire. How would you expect the solution to this problem to change? You don't have to do any math – just tell me what parts of the analysis would have to change and why.