Exam 2

Your Name:

Instructions

Solve each of the following problems to the best of your abilities. The exam is worth 100 points total and is calibrated for 90 minutes. Be sure to show all of your work for full credit.

Once you have completed the exam, hand it to me, and you can take a break before the second part of class. Class resumes at 10:00AM.

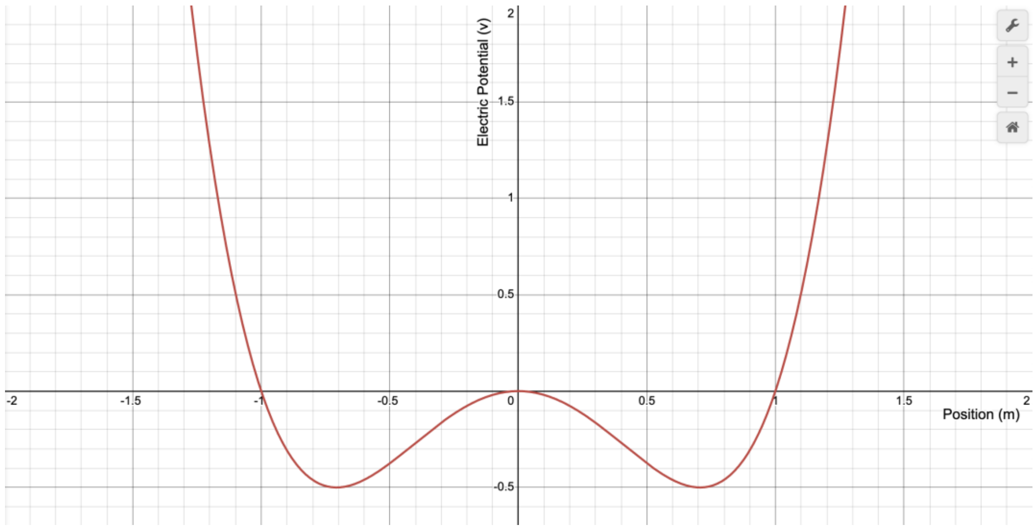
Good luck!

Problem 1

(25 points) The electric potential in a region of space is given by the equation:

$$V(x) = 2(x^4 - x^2)$$

The graph below shows the electric potential curve (plotted using Desmos).



a) (5 points) At which point(s) of space is the electric potential equal to zero? Explain your reasoning.
b) (5 points) At which point(s) of space is the electric field equal to zero? Explain your reasoning.
c) (5 points) Suppose I place a +0.50 mC charge at the position x = +1.00 meter. What is the force on the particle (magnitude and direction)?
d) (5 points) Suppose a +0.50 mC charge starts at position x = 2.00 m and moves to position x = 0.00 m. What is the change in potential energy of this particle?
e) (5 points) Suppose I were to introduce a DC offset of +5 V to the graph. In other words, I shifted the entire graph up by 5 V. How would that affect the change in potential energy on the particle that you calculated in part (d)? Explain your reasoning.

Problem 2

(25 points) A Helium-4 nucleus is launched with a velocity of $\vec{v} = 100\ \hat{x}\ \frac{m}{s}$ into a uniform magnetic field $\vec{B} = 0.5\ \hat{z}\ T$.

- The charge on a proton is $+1.6 \times 10^{-19}$ C.
- The charge on a neutron is 0.0 C (no charge).
- The mass of a proton approximately equals the mass of a neutron which is 1.67×10^{-27} kg.
- A Helium-4 nucleus consists of two protons and two neutrons.

a) (5 points) Sketch a diagram of the nucleus, the initial velocity, the magnetic field, and the path that the nucleus will trace as it moves through the field. Label the direction of the circular motion in your diagram.

b) (5 points) What is the speed of the helium nucleus after it completes 10 full revolutions in the field?

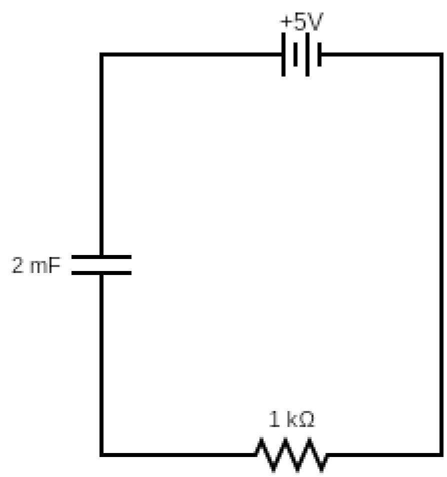
c) (5 points) Suppose I launched a Helium-3 nucleus into the magnetic field instead of a Helium-4 nucleus. Helium-3 consists of two protons and one neutron. Will the radius of the orbit be larger, smaller, or equal to that of the Helium-4 orbit? Why?

d) (5 points) Suppose I increased the strength of the magnetic field in this problem. Would you expect the radius of the path to increase, decrease, or stay the same? Why?

e) (5 points) Suppose I increased the strength of the magnetic field in this problem. Would you expect the frequency of the motion to increase, decrease, or stay the same? Why?

Problem 3

(25 points) An RC circuit is given in the diagram below. In it, a 5 V battery is connected in series to a 1000 Ω resistor and a 2 mF capacitor. You can assume that the capacitor starts off with no charge. The capacitor is allowed to charge in the circuit from time t = 0 seconds, onwards.



a) (5 points) What is the current in the circuit after a long time?
b) (5 points) What is the charge on the capacitor at time t = 3.5 seconds?
c) (5 points) Suppose I were to replace the resistor with a new one having double the resistance. Would you expect the current in the circuit at time t = 0 seconds to increase, decrease, or stay the same? Why?
d) (5 points) Suppose I were to replace the resistor with a new one having double the resistance. Would you expect the energy stored in the capacitor after it is fully charged to increase, decrease, or stay the same? Why?
e) (5 points) Suppose I were to replace the resistor with a new one having double the resistance. Would you expect the voltage across the capacitor after it is fully charged to increase, decrease, or stay the same? Why?

Problem 4

(25 points) A long, cylindrical wire with a radius of R carries a uniform change density of ρ (where ρ has units of charge per unit volume). We are going to calculate the electric field inside of the wire using Gauss' law.

$$\Phi_{Closed} = \frac{q_{enc}}{\epsilon_o}$$

a) (5 points) Sketch a diagram showing the wire and the Gaussian surface that you will use to calculate the electric field. We want to evaluate the electric field at a point a distance r from the center of the wire where r < R. Be sure to label any relevant dimensions of the Gaussian surface.
b) (5 points) The left side of Gauss' law (shown above) describes the net electric flux through the closed surface. Set up an expression for the net flux and simplify it using the diagram that you drew as a reference. Remember to think about symmetry to help you simplify the expression!
c) (5 points) The right side of Gauss' law (shown above) describes the net charge enclosed by the closed surface. Set up an expression for the net charge and simplify it using the diagram that you drew as a reference.
d) (5 points) Combine your answers for parts (b) and (c) to get an expression for the electric field inside of the wire.
e) (5 points) Suppose the charge density in the wire was not uniform. Instead, it had a dependence on the distance from center of the wire: $\rho(r) = Ar$ where A is some constant and r is the distance from the center of the wire. How would you expect the solution to this problem to change? You don't have to do any math – just tell me what parts of the analysis would have to change and why.