Exam 2

Your Name:

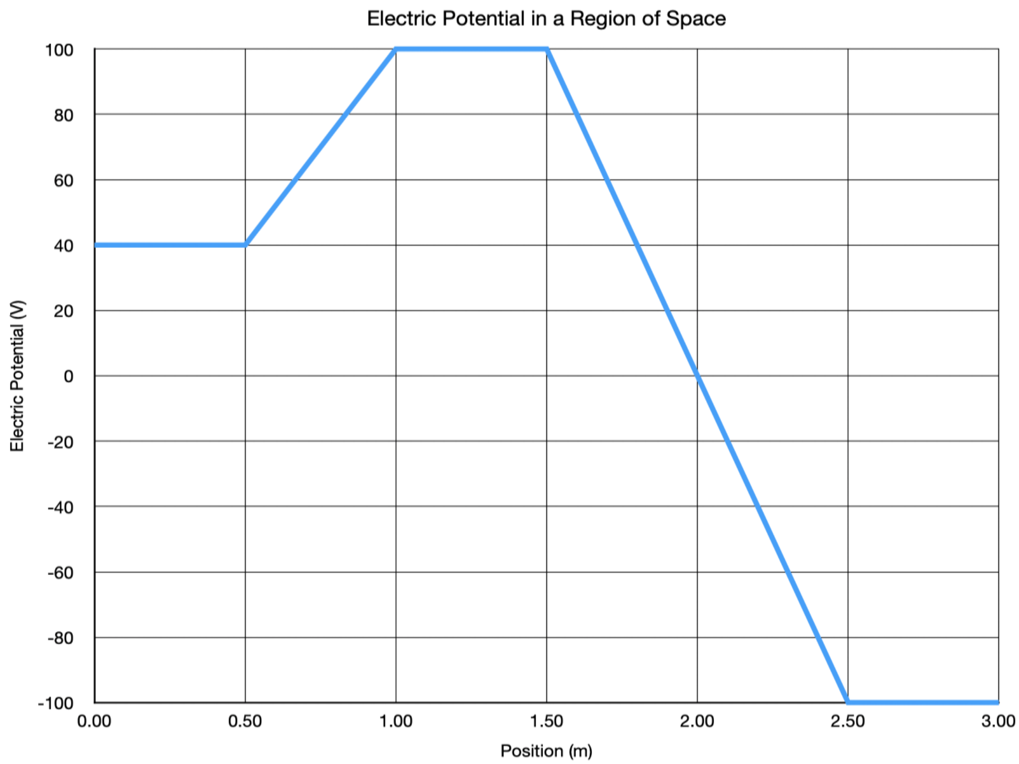
Instructions

Solve each of the following problems to the best of your abilities. The exam is worth 100 points total and is calibrated for 120 minutes. Once you have completed the exam, hand it to me, and you can take a break before the second part of class. Class resumes at 7:30PM.

Good luck!

Problem 1

(25 points) The electric potential in a region of space is given in the graph below.



a) (5 points) In which region(s) of space is the electric potential equal to zero? Explain your reasoning.

Since this is a graph of electric potential vs. time, you can just read off the value of electric potential. Thus, the potential is zero at the position x = 2.00 m.

b) (5 points) In which region(s) of space is the electric field equal to zero? Explain your reasoning.

The electric field is the negative gradient of the potential. Thus, the field will be zero when the slope of the graph is equal to zero. This occurs from x = 0.00 - 0.50 m, x = 1.00 - 1.50 m, and x = 2.50 - 3.00 m.

c) (5 points) Suppose I place a -0.57 mC charge at the position x = 2.00 meters. What is the force on the particle (magnitude and direction)?

The electric field is given by the negative slope of the graph. The slope of the graph at that position is rise over run or –200 V / 1.00 m. Taking the negative of that answer yields +200 V/m. The force on the particle is $\vec{F} = q\vec{E} = -0.114\ N$. Since the sign is negative, the direction is the -x direction.

d) (5 points) Suppose I place the –0.57 mC charge at position x = 0.00 m and drag it to position x = 3.00 m. What is the net work that I do as I move this particle?

The total change in potential between x = 0.00 m and x = 3.00 m is –100 V – 40 V = -140 V. The change in potential energy is $\Delta U = q\Delta V = 0.0798\ J$. This means that the work done by the field is –0.0798 J, but the work done by me as I move the particle is 0.0798 J. In other words, I have to do positive work to force the particle to move to lower potential since it is negatively charged.

e) (5 points) Suppose I were to introduce a DC offset of +100 V to the graph. In other words, I shifted the entire graph up by 100 V. How would that affect the force on the particle from part (c)? Explain your reasoning.

Electric field depends on the slope of the graph – not its absolute value. Thus, a DC offset would not affect the field, meaning the force would also stay unchanged.

Problem 2

(25 points) Two Helium-4 nuclei in a particle collider are fired towards one another on a collision course. They start very apart from each other and are each given an initial velocity of 10,000 m/s.

- The charge on a proton is $+1.6 \times 10^{-19}$ C.
- The charge on a neutron is 0.0 C (no charge).
- The mass of a proton approximately equals the mass of a neutron which is 1.67×10^{-27} kg.
- A Helium-4 nucleus consists of two protons and two neutrons.

a) (5 points) What is the minimum distance that the two nuclei reach before reversing direction?

We can start with conservation of energy:

$$\Delta U = \Delta K$$

$$\frac{kq_1q_2}{r} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\frac{kq^2}{d} = mv^2$$

$$d = \frac{kq^2}{mv^2}$$

The total charge on one of the nuclei is two times the charge on a proton since there are two protons per nucleus. The total mass of one of the nuclei is four times the mass of a proton / neutron since there are four nucleons per nucleus. Plugging in all of the values yields:

$$d = 1.4 \times 10^{-9}\, m$$

b) (5 points) What is the electric potential energy of the system when the nuclei are at the minimum distance from part (a)?

Electric potential is not the same as the electric potential energy. The electric potential energy of the system at the minimal distance is just the sum of the kinetic energies (i.e. $U = mv^2$). This comes from the law of conservation of energy.

$$U = 6.68 \times 10^{-19}\, J$$

c) (5 points) Would the minimum distance increase, decrease, or stay the same if I used Helium-3 instead of Helium-4? Helium-3 is a helium nucleus with two protons and one neutron. Explain your reasoning.

If you used Helium-3 nuclei, the mass of the particles would decrease while everything else stays constant. According to the equation we derived in part (a), the distance would increase since it is inversely proportional to mass.

d) (5 points) Would the minimum distance increase, decrease, or stay the same if I fired them at each other with a slower initial speed? Explain your reasoning.

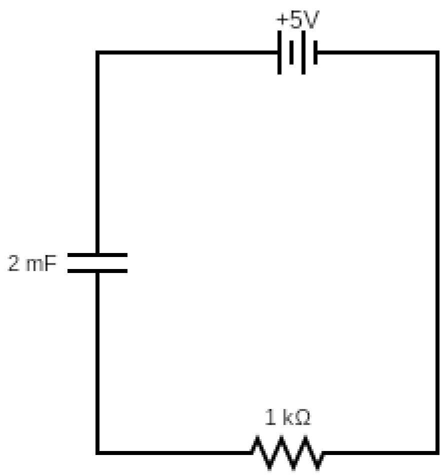
According to the equation we derived in part (a), the distance would increase since it is inversely proportional to initial speed.

e) (5 points) Is the electric force on each helium nucleus constant or changing as they move towards each other in the particle collider? Explain your reasoning.

The force is given by Coulomb's law. Since the distance between the particles changes as they approach each other, the force changes as well.

Problem 3

(25 points) An RC circuit is given in the diagram below. In it, a 5 V battery is connected in series to a 1000 Ω resistor and a 2 mF capacitor. You can assume that the capacitor starts off with no charge on it at time t = 0 seconds.



a) (5 points) What is the current in the circuit at time t = 0 seconds?

The capacitor acts like a wire at this instant in time. Thus, I = V/R = 0.005 A

b) (5 points) What is the current in the circuit at time t = 2.5 seconds?

$$I(t) = I_{Max}e^{-\frac{t}{RC}}$$

$$I(t) = 0.005e^{-\frac{t}{2}}$$

$$I(2.5) = 0.005e^{-\frac{2.5}{2}} = 0.0014\,A$$

c) (5 points) What is the current in the circuit after a long time?

The capacitor acts like a break at this instant in time. Thus, I = 0 A.

d) (5 points) What is the charge on the capacitor after a long time?

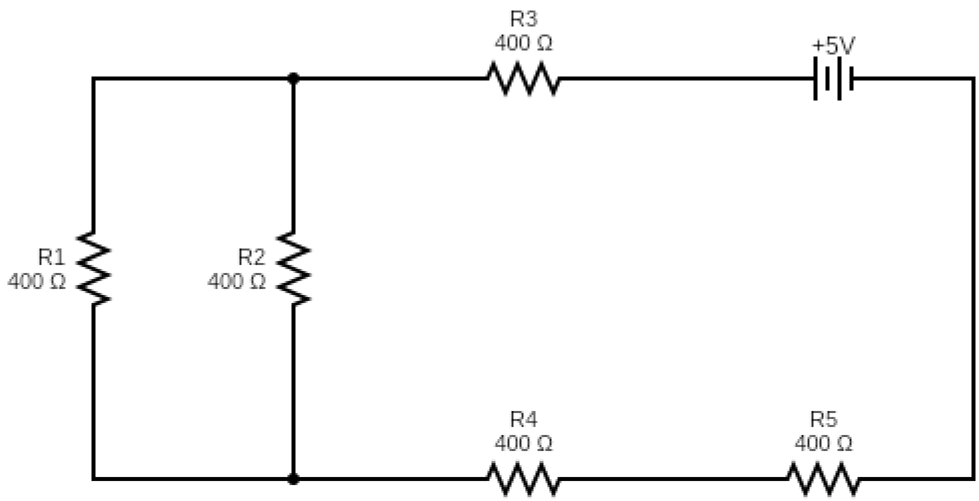
The charge on the capacitor is just Q = CV after a long time => (2 mF)(5 V) = 10 mC

e) (5 points) Suppose I were to increase the capacitance of the capacitor to 4 mF. Would you expect the current in the circuit at time t = 0 seconds to increase, decrease, or stay the same? Why?

Since the capacitor acts like a wire at time t = 0 seconds, increasing the capacitance would have no effect on the initial current.

Problem 4

(25 points) Five resistors, each with a resistance of 400 Ω, are connected to a 5 V battery as shown in the diagram below.



a) (5 points) What is the current drawn from the battery?

The total resistance of the circuit can be found by (1) adding R1 and R2 in parallel and (2) adding the resulting resistor to all of the others in series. Then, you can use Ohm's law to find the current: I = (5 V) / (1400 Ω) = 0.00357 A.

b) (5 points) What is the total power generated by the battery?

$$P = IV$$

$$P = (0.00357\,A)(5\,V)$$

$$P = 0.0179\,W$$

c) (5 points) Suppose I were to decrease the value of resistor R3 to 200 Ω, keeping everything else equal. Would the current drawn from the battery increase, decrease, or stay the same? Why?

This would reduce the total resistance of the circuit. Since V = IR, reducing the resistance will increase the current (assuming the battery stays constant).

d) (5 points) Suppose I were to increase the voltage of the battery to 10 V, keeping everything else equal. Would the current drawn from the battery increase, decrease, or stay the same? Why?

Again, by Ohm's law, we have V = IR. Increasing the voltage will increase the current (assuming resistance stays the same).

e) (5 points) Suppose I were to accidentally drop a bare wire onto the circuit so that resistor R1 is shorted (i.e. R1 is effectively equal to 0 Ω), keeping everything else equal. What is the new current drawn from the battery?

The new resistance of the circuit is just R3, R4, and R5 in series since the short across R1 also shorts R2. Thus, I = (5 V) / (1200 Ω) = 0.00417 A