Exam 1

Your Name:

Instructions

Solve each of the following problems to the best of your abilities. The exam is worth 100 points total and is calibrated for 120 minutes. Once you have completed the exam, hand it to me, and you can take a break before lab. Lab starts at 7:30PM.

Good luck!

Problem 1

(20 points) A positive point charge (q = +2.5 mC) is placed on the y-axis at y = +4.5 cm. A negative point charge (Q = -2.5 mC) is placed on the x-axis at x = +4.0 cm.

a) (4 points) What is the force on the negative point charge (Q) from the positive point charge (q) in this configuration? Be sure to include both the magnitude / direction or give me the components of the force.

We can use Coulomb's law to calculate the electric force on the negative point charge (Q). You can either find the magnitude of the force and then calculate the angle using trigonometry or use the vector formulation for Coulomb's law. I will use the vector formulation here.

We start off by defining the positions of each charge and the relative position between them:

$$\overrightarrow{r_1} = 0.045\,\hat{y}$$

$$\overrightarrow{r_2} = 0.04\,\hat{x}$$

$$\overrightarrow{r_{12}} = \overrightarrow{r_2} - \overrightarrow{r_1} = 0.04\,\hat{x} - 0.045\,\hat{y}$$

$$r_{12}^2 = (0.04)^2 + (-0.045)^2 = 0.003625$$

Now, we can use that information to solve for the force:

$$\overrightarrow{F_{12}} = \frac{kq_1q_2}{r_{12}^2}\widehat{r_{12}}$$

$$\overrightarrow{F_{12}} = \left[\frac{(9 \times 10^9\,Nm^2/C^2)(2.5 \times 10^{-3}C)(-2.5 \times 10^{-3}C)}{0.003625\,m^2}\right]\left[\frac{0.04\,\hat{x} - 0.045\,\hat{y}}{\sqrt{0.003625}}\right]$$

$$\overrightarrow{F_{12}} = (-15517241.3793)(0.6644\,\hat{x} - 0.7474\,\hat{y})$$

$$\overrightarrow{F_{12}} = -10309094\,\hat{x} + 11597730\,\hat{y}\,N$$

If instead, you decided to solve for the magnitude of the force and then find the angle via trigonometry, you would have gotten:

$$F_{12} = 15517240.7\ N$$

$$\tan^{-1}\left(\frac{0.045}{0.04}\right) = 0.844\ rad$$

$$\theta = 2.3\ rad$$

b) (4 points) Consider the force on the positive point charge (q) from the negative point charge (Q) in this configuration. How does this force compare to what you calculated in part (a)?

The force is equal in magnitude and opposite in direction.

c) (4 points) How do your answers in parts (a) and (b) relate to Newton's laws? Hint: consider Newton's third law.

Newton's third law tells us that for every action there is an equal and opposite reaction. The forces between the two point charges are equal and opposite, as per Newton's law.

d) (4 points) Suppose I connect the two point charges together with a rigid rod, creating an electric dipole. What is the dipole moment of the two charges? Be sure to include both the magnitude / direction or give me the components of the dipole moment.

The electric dipole moment of the system is given by $\vec{p} = q\vec{d}$. Plugging in the charge and displacement yields:

$$\vec{p} = (2.5 \times 10^{-3}\ C)(-0.04\ m\,\hat{x} + 0.045\ m\,\hat{y})$$

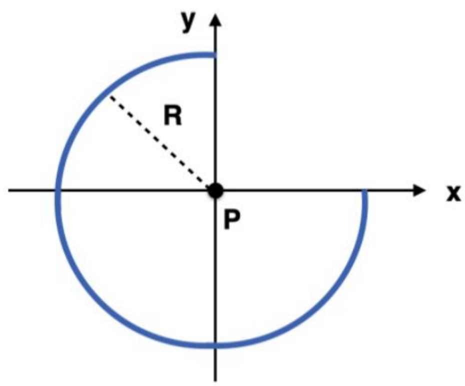
$$\vec{p} = -0.0001\,\hat{x} + 0.0001125\,\hat{y}\ \ C \cdot m$$

e) (4 points) If I were to decrease the distance between the two point charges, would the magnitude of the dipole moment increase, decrease, or stay the same? Why?

The dipole moment would decrease since the moment is directly proportional to the distance between the charges.

Problem 2

(30 points) Consider a thin ring of charge that starts at 90 degrees and extends ¾ of the way around a full circle. The ring has a radius of R and a total charge Q. Our goal is to find the electric field at the center of the ring (point P) using Coulomb's law for continuous charge distributions:



$$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$$

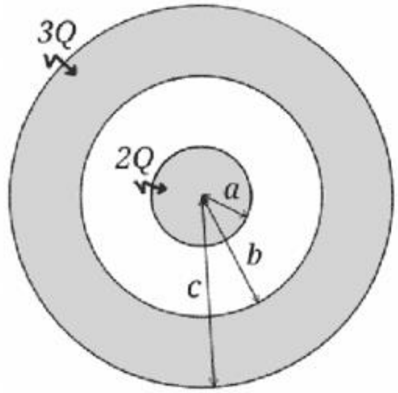
Parts a – e

You could set up this problem as it stands and integrate over ¾ of the ring of charge. However, the electric field from the ring in the region where x < 0 and y > 0 cancels the electric field from the ring in the region where x > 0 and y < 0 due to symmetry. Thus, you could just analyze the electric field from a ¼ ring rather than a ¾ ring. We did this problem together in class, so check out your notes!

(5 points) Suppose that instead of having a uniform charge density, the ring had a charge density given by $\lambda(\theta) = \sin\theta$. In other words, the charge density changes depending on which angle you are at on the ring. What would change in the steps taken above? Note: you don't have to actually calculate this integral – just explain to me what would change with this varying charge density.

The tiny bit of charge dQ would now include a non-constant charge density that depends on the sine of the angle around the loop. Everything else should stay the same.

Problem 3



(20 points) An insulating sphere with a radius of a is given a charge of +2Q. A conducting shell with an inner radius of b and an outer radius of c is given a charge of +3Q.

a) (4 points) What is the charge on the inner surface of the conducting shell (r = b)?

The charge on the inner surface is equal in magnitude and opposite in sign to the charge of the insulating ball. It is –2Q.

b) (4 points) What is the electric field outside of the conducting shell (r > c)?

The total enclosed charge by a spherical Gaussian surface which is located outside of the shells is 3Q + 2Q = 5Q. As we saw in class, after applying Gauss' law, we get:

$$\vec{E} = \frac{5kQ}{r^2}\hat{r}$$

c) (4 points) What is the electric field inside of the insulating sphere (r < a)?

Check out example 22.13 in your book.

d) (4 points) Suppose I were to decrease radius of the insulting sphere (radius a) while keeping the charge on it constant, would the charge on the inner surface of the conducting shell increase, decrease, or stay the same? Why?

The net charge on the inner surface would stay the same. If you were to draw a Gaussian surface within the conducting shell, you will notice that the net charge consists of the charge on the insulating sphere and the charge on the inner surface. The net flux / electric field inside of the conductor must be zero at equilibrium. Thus, the net flux does not depend on the configuration of the charges – since the amount of charge on the sphere does not change, the charge on the inner surface does not change.

e) (4 points) Suppose I were to shift the insulating sphere slightly off center, so that it was a little closer to the right side of the conducting shell than the left. What can we now conclude about the electric field inside of the conducting shell? Be sure to explain your reasoning.

The charge distribution on the inner surface of the conducting shell would be non-uniform since we have lost symmetry. However, the net electric field within the conductor would still be zero. It is a conductor at equilibrium, thus the electric field inside of it must be zero.

Problem 4

(10 points) What are the steps taken to charge an object by induction?

The steps are described in your book in section 21.2.

Problem 5

(10 points) Can electric field lines cross each other at some point in space? Why or why not?

No, they cannot. If they were to cross, they must add together as vectors. Otherwise, you would have two different electric forces acting on a charge particle at that point.

Problem 6

(10 points) In your reading, you found that excess charge on a conductor resides on its surface (not within the conductor itself). Why is this the case? Hint: you can make an argument for this using Gauss' law.

I was looking for two specific facts when answering this problem:

1. Charges are free to move inside of a conductor
2. The net flux on a Gaussian surface drawn within a conductor must be zero, otherwise there would be a net electric field.

A net field would force charges to move until they reach equilibrium.